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Analysis of Cracked, Adhesively Bonded Laminated Structures

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Techniques to analyze primarily two-ply, adhesively bonded structures are discussed. Finite-element and mathematical methods of analysis for determining stress intensity factors are outlined. The analytical stress intensity factors obtained are compared with experimental results. A method to account for the influence of out-of-plane bending on stress intensity factors obtained for various panel and crack geometries are given. The influence of debond size, adhesive, and adherend properties on analytical stress intensity factors is investigated.

Introduction

ADHESIVE bonding has features that make it attractive for use in aircraft structural applications. The improved fracture behavior of metal laminates is reported by Kaufman[1] and Alic and Chang.[2] The increased fatigue crack growth life of adhesively bonded laminated structures compared to monolithic structures is demonstrated in Refs. 3-6. Mainly, experimental results are presented in these references and no analytical techniques were developed. In Refs. 7 and 8, the finite-element analysis method is used to analyze bonded structures. An attempt was made to correlate analytical debond propagation in the adhesive with experimental results.[8] The problem of debond propagation in metal-to-composite bonded structures was investigated in Ref. 9.

Several investigators have studied the problems of cracked, adhesively bonded structures.[7,8,10-13] In Refs. 10-12, a complex variable approach was used to analyze the problem. In Ref. 13, the problem of a cracked plate bonded to an uncracked plate was solved using the Fourier transform technique and reducing it to the solution of an integral equation. A finite-element approach was used to study the problem in Refs. 7 and 8. In these studies, however, no attempt was made to correlate the experimental crack growth data with the analytical stress intensity factors; hence, these methods need to be investigated in order to develop confidence in damage-tolerant analytical methods for bonded structures. The results, abstracted from a broader study,[14,15] designed to develop and verify analytical techniques for predicting crack growth behavior in bonded structures, are discussed here.

Finite-Element Method of Analysis

A cracked, adhesively bonded structure represents a three-dimensional layered structure and a rigorous mathematical or finite-element analysis represents several degress of complexity; thus, certain simplifying assumptions have to be made. These assumptions should be such that they do not sacrifice the accuracy of the analysis. The assumptions made in this analysis are:

1) Each layer is considered as a two-dimensional structure under a state of plane stress.

2) The thickness h_a of the adhesive is small compared to the thickness of the plates. Therefore, the adhesive may be treated as an elastic shear spring rather than as an elastic continuum.[16] The continuous shear spring is replaced by shear elements.

3) The layers are connected by shear elements representing the adhesive.

4) The bending stiffness of the cracked and sound layers is negligible.

Consider the two-ply, adhesively bonded structure shown in Fig. 1, with a through-crack in the plate and a debond in the adhesive around the crack in material 1, no crack in material 2. In this figure, the debond is shown to be longer than the crack. This will be the case when the initial debond in the adhesive is produced during the manufacturing process and a subsequent in-service flaw initiates in the plate. If there is no initial debond in the adhesive, the length of the debond will be the same length as the crack. Experiments and analyses[10,14,15] confirm that this will generally be the case when the initial flaw has grown, due to fatigue with no previous debonding in the adhesive, and the edge of the debond will propagate with the crack tip.

The bonded plates of both materials are modeled as rectangular or triangular elements, as shown in Fig. 2. A cracked element is provided ahead of the crack tips in the cracked layer. The x and y coordinates of the grid points in the bonded regions of the two plates are kept the same, i.e., the finite-element model in the bonded regions of the plates is identical.

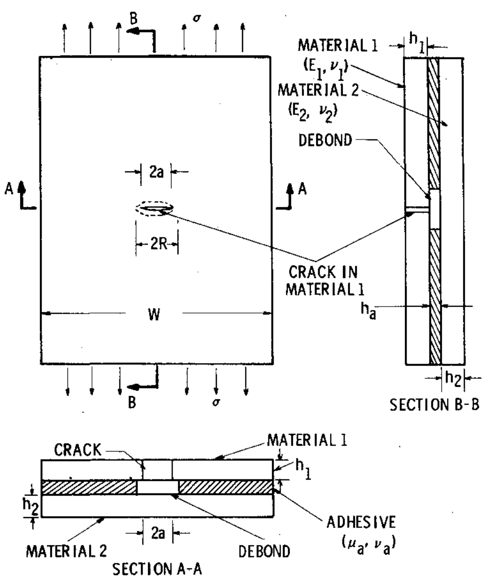

Fig. 1 Two-ply, adhesively bonded structure with a center-crack in one ply.

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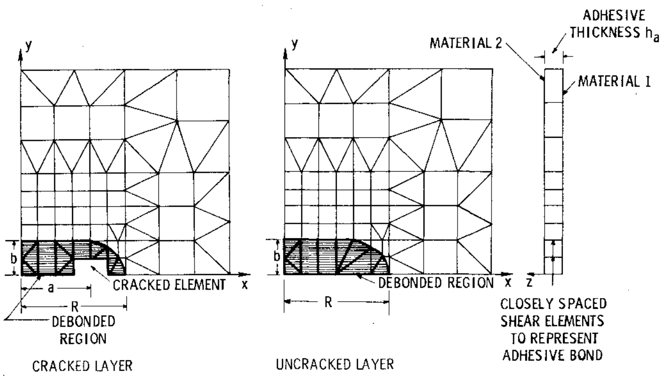

Fig. 2 Method of finite-element modeling for adhesively bonded structure.

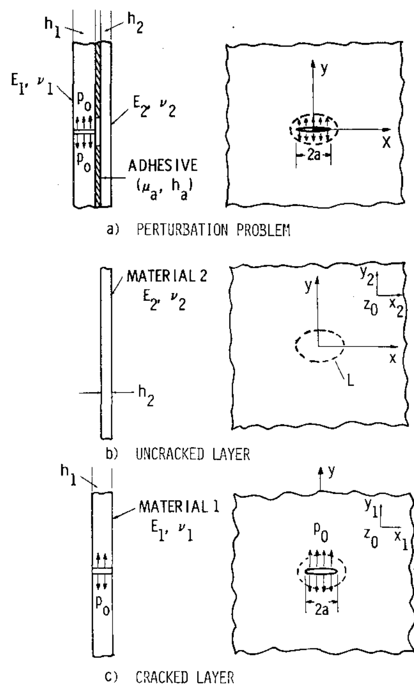

Fig. 3 Perturbation problem for cracked, adhesively bonded structure.

The adhesive is subjected to high shear stresses around the periphery of the debond in the bonded region. A closely spaced mesh is provided to give an accurate determination of shear stresses and load transfer in the uncracked layer, resulting in more accurately computed stress intensity factors. In the bonded region, the grid points of the two materials are connected by rectangular, prismatic-shaped shear elements that have the same material properties as the adhesive. The thickness of the shear elements perpendicular to the cracked-uncracked plies is taken as the thickness of the adhesive.

The finite-element analysis in this study was accomplished using the NASTRAN computer code (MSC version). The cracked element developed at MIT[17] and incorporated in NASTRAN at Northrop, was used.

Mathematical Methods of Analysis

Mathematical methods of analysis are very useful in analyzing crack problems in bonded structures. This is particularly so where boundaries are regular and the crack lengths are small compared to the dimensions of the structure. A complex variable approach[10-12,14,15] to reduce the problem of cracked structure to the solution of integral equations is considered convenient and is used in the present investigations. The assumptions outlined for the finite-element method are also made in the mathematical method. In Ref. 10, the problem of a cracked, metallic layer adhesively bonded to a composite (orthotropic) layer is considered. Using a similar approach, the problem of a cracked, metallic layer bonded to another metallic layer (Fig. 1) is formulated, assuming the plates to be infinitely wide. The problem of Fig. 1 is reduced to the solution of the perturbation problem shown in Fig. 3a. This problem will be looked on as consisting of plate 2 with body forces in region D (Fig. 3b), and plate 1 with body forces and loading on the crack surface, as shown in Fig. 3c.

Let u_1, v_1 and u_2, v_2 be the x,y components of the in-plane displacement vectors in materials 1 and 2, respectively, and τ_x and τ_y the components of the shear stress acting on the adhesive. Assumption 2, gives the following continuity conditions:

$$u_1 - u_2 = (h_a/\mu_a)\tau_x$$

$$v_1 - v_2 = (h_a/\mu_a)\tau_y \tag{1}$$

where μ_a is the shear modulus of the adhesive. The body forces acting on plates 1 and 2 (Fig. 3) are given by

$$X_1 = -\frac{\tau_x}{h_1}, \quad Y_1 = -\frac{\tau_y}{h_1}, \quad X_2 = \frac{\tau_x}{h_2}, \quad Y_2 = \frac{\tau_y}{h_2} \tag{2}$$

Displacements in the Uncracked Plate

The displacements in the uncracked plate under body forces X_2, Y_2, acting at $z_0\,(x_0,y_0)$ are given by [15,18]

$$2\mu_2(u_2 + iv_2) = -\kappa S[\log(z - z_0) + \log(\bar{z} - \bar{z}_0)]$$

$$+\bar{S}\left[\frac{z - z_0}{\bar{z} - \bar{z}_0} - 1\right] - \bar{S}\left[\frac{z + z_0}{\bar{z} + \bar{z}_0} - 1\right] \tag{3}$$

Assuming that the body forces X_2, Y_2 are continouus functions of z_0 or (x_0, y_0), as defined in region D, the displacements at point $z = x + iy$ in the plane are given by

$$u_2(x,y) = \int\int_D [K_{11}(x,y;\, x_0,y_0)X_2(x_0,y_0)$$

$$+K_{12}(x,y;\, x_0,y_0)\,Y_2(x_0,y_0)]\,dx_0dy_0$$

$$v_2(x,y) = \int\int_D [K_{21}(x,y;\, x_0,y_0)X_2(x_0,y_0)$$

$$+K_{22}(x,y;\, x_0,y_0)\,Y_2(x_0,y_0)]\,dx_0dy_0 \tag{4}$$

where the kernels $K_{ij}\,(i=1,2;\ j=1,2)$ are given by Green's functions.[15]

The displacements in the cracked plate will consist of two components, u_{11}, v_{11}, due to uniformly applied pressure on the crack surface[19] and u_{12}, v_{12}, due to body forces acting on the plate surface. Displacements u_{11} and v_{11} are given by

$$u_{11}(x,y) = \frac{p_0}{4\mu_1}\Big\{(\kappa - 1)Re[(z^2 - a^2)^{1/2}]$$

$$-2yIm\left[\frac{z}{(z^2 - a^2)^{1/2}}\right] + (1-\kappa)x\Big\} = p_0 f_1(x,y) \tag{5a}$$

$$v_{11}(x,y) = \frac{p_0}{4\mu_1}\Big\{(\kappa + 1)Im[(z^2 - a^2)^{1/2}]$$

$$-2yRe\left[\frac{z}{(z^2 - a^2)^{1/2}}\right] + (1-\kappa)y\Big\} = p_0 f_2(x,y) \tag{5b}$$

where $\kappa=(3-\nu_1)/(1+\nu_1)$. The displacements in the cracked plate due to body forces X_1 and Y_1 can be written in a form similar to Eqs. (5) and are given as

$$u_{12}(x,y)=\int\int_D [H_{11}(x,y;x_0,y_0)X_1(x_0,y_0)+H_{12}(x,y;x_0,y_0)Y_1(x_0,y_0)]\,dx_0dy_0 \quad (6a)$$

$$v_{12}(x,y)=\int\int_D [H_{21}(x,y;x_0,y_0)X_1(x_0y_0)+H_{22}(x,y;x_0,y_0)Y_1(x_0,y_0)]\,dx_0dy_0 \quad (6b)$$

where $H_{ij}(i=1,2;\ j=1,2)$ are given in Refs. 10 and 15. The total displacements in the base plate of material 1 may then be obtained by adding Eqs. (5) and (6).

Using displacements u_1, u_2, v_1 and v_2 in Eq. (1) and making use of Eq. (2), the following system of integral equations for the unknown shear stresses τ_x and τ_y is obtained.

$$\frac{h_a}{\mu_a}\tau_x(x,y)+\int\int_D [R_{11}(x,y;x_0,y_0)\tau_x(x_0,y_0)+R_{12}(x,y;x_0,y_0)\tau_y(x_0,y_0)\,dx_0dy_0=p_0f_1(x,y) \quad (7a)$$

$$\frac{h_a}{\mu_a}\tau_y(x,y)+\int\int_D [R_{21}(x,y;x_0,y_0)\tau_x(x_0,y_0)+R_{22}(x,y;x_0,y_0)\tau_y(x_0,y_0)]\,dx_0dy_0=p_0f_2(x,y) \quad (7b)$$

The functions f_1, f_2 are given by Eqs. (5) and

$$R_{11}=\frac{H_{11}}{h_1}+\frac{K_{11}}{h_2},\qquad R_{12}=\frac{H_{12}}{h_1}+\frac{K_{12}}{h_2}$$
$$R_{21}=\frac{H_{21}}{h_1}+\frac{K_{21}}{h_2},\qquad R_{22}=\frac{H_{22}}{h_1}+\frac{K_{22}}{h_2} \quad (8)$$

The kernels $R_{ij}(i,j=1,2)$, which have logarithmic singularities, are known and are square integrable in region D.

Stress Intensity Factors

The cleavage component k_1 of mode I stress intensity factor $(k_1=k/\sqrt{\pi})$ is determined by adding the effects of the crack surface pressure p_0, and the body forces X_1 and Y_1.

$$k_1=p_0\sqrt{a}+\int\int_D [q_1(x_0,y_0)X_1(x_0,y_0)+q_2(x_0,y_0)Y_1(x_0,y_0)]\,dx_0dy_0 \quad (9)$$

where $q_1(x_0,y_0)$ and $q_2(x_0,y_0)$ are the cleavage components of the stress intensity factor due to the concentrated body forces X_1 and Y_1, respectively, and can be obtained from Ref. 15.

The system of integral equations given by Eqs. (7) are the Fredholm type and are solved by using standard numerical techniques.[10,15] A generalized program to obtain stress intensity factors cannot be developed, as convergence of the solution is dependent on the crack length and the thickness and material properties of adherends and adhesive. Two computer programs, one for small crack lengths ($a\leq 10$ mm) and the other for larger crack lengths ($12.5\leq a\leq 25$ mm) have been written to compute stress intensity factors.[15]

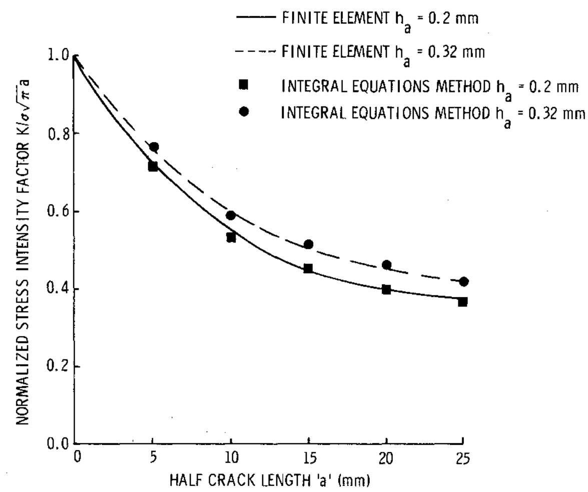

Fig. 4 Comparison of finite-element determined stress intensities with integral equation solution (two-layer, center-cracked, bonded structure).

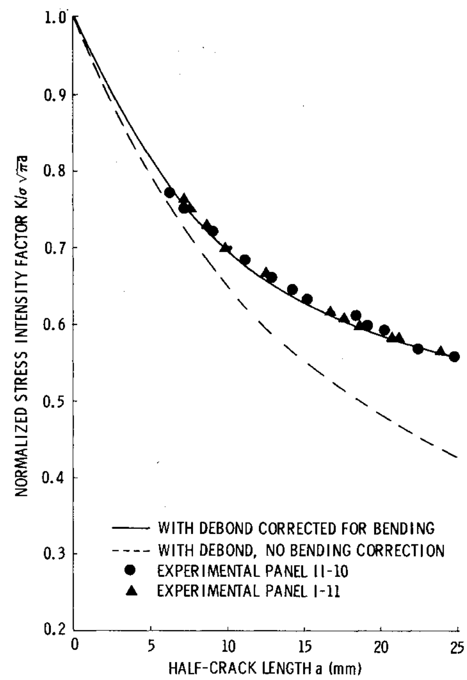

Fig. 5 Comparison of analytical and experimental stress intensity factors for a two-ply, cracked, adhesively bonded panel.

Comparison of Mathematical and Finite-Element Methods

The two-ply, adhesively bonded structure shown in Fig. 1 was analyzed using finite-element and mathematical methods. The results obtained by the two methods, shown in Fig. 4, are within 2%. The computer run time (IBM 370 computer) for finite-element analysis is about 7 min, and for mathematical methods, the computer run times are 50 s for small crack length programs, and 1-2 min for long crack length programs, depending on the debond size.[15]

Comparison of Analytical and Experimental Stress Intensity Factors

Figure 5 shows the experimentally obtained stress intensity factors from crack growth analysis for two two-ply, center-cracked, adhesively bonded test panels (7075-T73 aluminum, with a thickness of 1.6 mm per ply, 150 mm wide, with FM 73 adhesive, 0.2 mm thick). The figure also shows analytical stress intensity factors obtained by finite-element analysis,

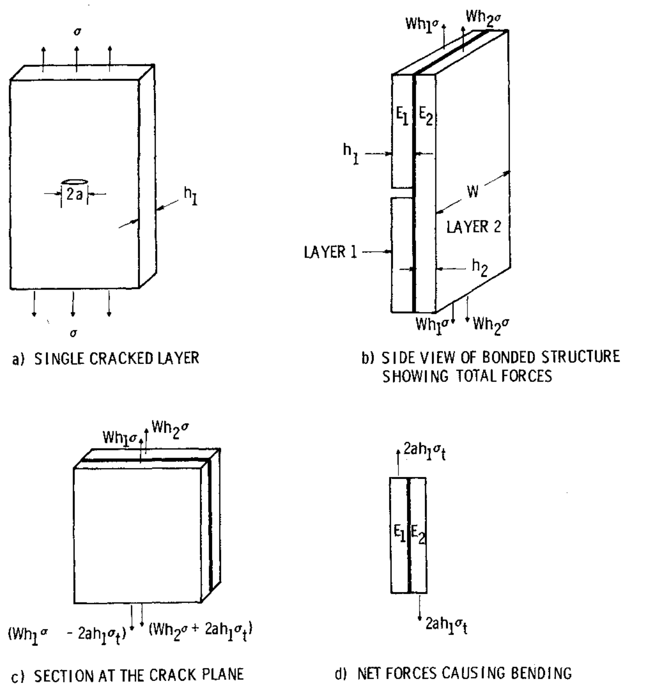

Fig. 6 Bending correction for adhesively bonded panel with a crack in one ply.

assuming an elliptical debond in the adhesive, with a minor axis to major axis ratio of 0.1 (debond shape and size observed in the experiment).[14] It is seen that the finite-element, stress intensity factors are considerably lower than these obtained experimentally.

Influence of Out-of Plane Bending

The finite-element and mathematical methods for the analysis of cracked, adhesively bonded structures previously discussed are for the extensional type of loading, and it is assumed that both the cracked and sound layers have no bending stiffness. The presence of a crack in one layer of a bonded structure will give rise to out-of-plane bending due to lack of symmetry caused by the presence of a crack. This out-of-plane bending will increase the stress intensity factors. This influence of bending (increase in stress intensity factors) in a structure will increase as the crack length increases. This increase in the stress intensity factors will affect the crack propagation rates and should be accounted for in the analysis.

In a single, cracked layer (Fig. 6a), the force released at the crack plane due to the presence of a crack is entirely taken by the stress singularities ahead of the crack tips. When such a cracked layer is part of an adhesively bonded structure (Fig. 1), the same force (area times stress $=2ah_l\sigma$) release at the crack plane is taken partly by the cracked layer in the form of stress singularities similar to a single cracked layer, and the remainder of the force is transmitted to the adjacent sound layer through the adhesive. Let the force transmitted to the sound layer be $2ah_l\sigma_t$ (area times stress) where σ_t is the stress transferred to the sound layer. The total force acting on the end of each layer is shown in Fig. 6b. Due to the force transferred to the sound layer, the force in each plate at the plane of the crack will be different from that at the ends, as shown in Fig. 6c. The net internal unbalanced force between the two layers at the crack plane is equal to that which is transferred from the cracked to the sound layer, i.e., $2ah_l\sigma_t$. This unbalanced force causes the bending (Fig. 6d).

Neglecting adhesive thickness, the couple generated by the load transfer between the two plies is given by

$$C=2ah_l\sigma_t\frac{h_l+h_2}{2}=ah_l(h_l+h_2)\sigma_t \qquad (10)$$

Assuming that the bending is resisted by the entire plate width (W), the maximum bending stress in the layer is given by

$$\sigma_b=\frac{Cy_{\max}}{I}=\frac{ah_l(h_l+h_2)\sigma_t}{I}y_{\max} \qquad (11)$$

where $y_{\max}$ is the distance of the extreme fibers of the cracked plate from the neutral axis, and I is the moment of inertia of the section. For computing the values of $y_{\max}$ and I, the thickness of the adhesive may be neglected and the calculations based on the the initially uncracked cross section of both plies.

Load Transfer Factor

Define the term "load transfer factor" with the symbol M as the ratio of the load transferred (to the sound layer) to the total load released due to the presence of the crack.

$$M=(2ah_l\sigma_t/2ah_l\sigma)=(\sigma_t/\sigma) \qquad (12)$$

Substituting Eq. (12) in Eq. (11) gives

$$\frac{\sigma_b}{\sigma}=\frac{ah_l(h_l+h_2)}{I}My_{\max}=BC,\ \text{bending correction factor} \qquad (13)$$

Maximum bending stress is taken in computing the bending correction as this represents the worst case.

The Concept of Effective Stress

In a cracked, finite width, single-layer structure (Fig. 6a), the stress intensity factor is given as

$$K_s=\sigma\sqrt{\pi a}\,f(a/W) \qquad (14)$$

where $f(a/W)$ is the finite width correction.

Let K_A be the stress intensity factor in a cracked, adhesively bonded structure with half-crack length a and subjected to an applied stress of σ (Fig. 1), obtained by the finite element or mathematical approaches discussed earlier. The effective stress σ_e (function of crack length), acting remotely on a single-layer structure such as shown in Fig 6a, and giving the stress intensity factor K_A at half-crack length a, is given by

$$K_A=\sigma_e\sqrt{\pi a}\,f(a/W) \qquad (15)$$

using Eq. (14), K_A can be expressed as

$$K_A=\sigma_e(K_s/\sigma),\ \text{or},\ \sigma_e=(K_A/K_s)\sigma \qquad (16)$$

Interrelation Between Load Transfer Factor and Stress Intensity Factors

The stress transferred to the sound layer is the difference between the remotely applied stress and the uniform in-plane stress that matches the required singularity effect in the cracked layer, i.e.,

$$\sigma_t=\sigma-\sigma_e=\sigma-(K_A/K_s)\sigma \qquad (17)$$

Substituting Eq. (17) in Eq. (12) gives

$$M=\sigma_t/\sigma=1-(K_A/K_s) \qquad (18)$$

K_A is obtained from the finite-element analysis of cracked adhesively bonded structure and K_s is available in the form of Eq. (14), (Refs. 20 and 21). The value of M obtained here is used in computing the bending correction in Eq. (13). The stress intensity factor that includes the influence of bending is given by

$$K_A^*=(1+BC)K_A \qquad (19)$$

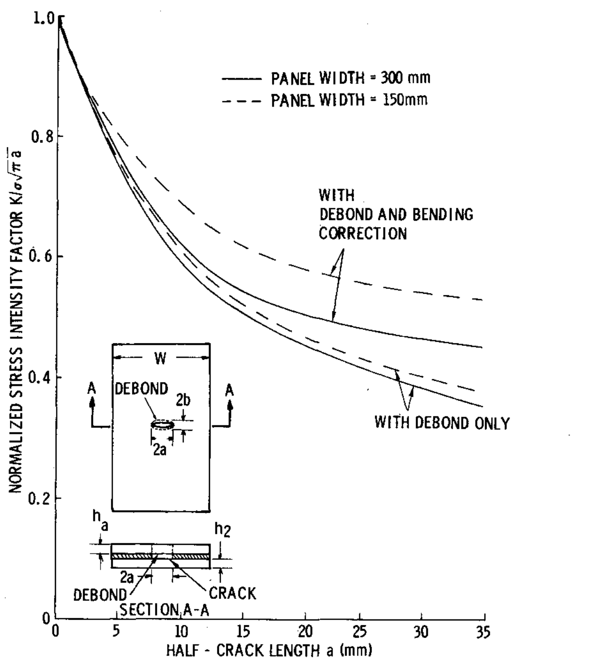

Fig. 7 Variation of stress intensity factors as a function of a half-crack length for a two-ply bonded structure with a center crack.

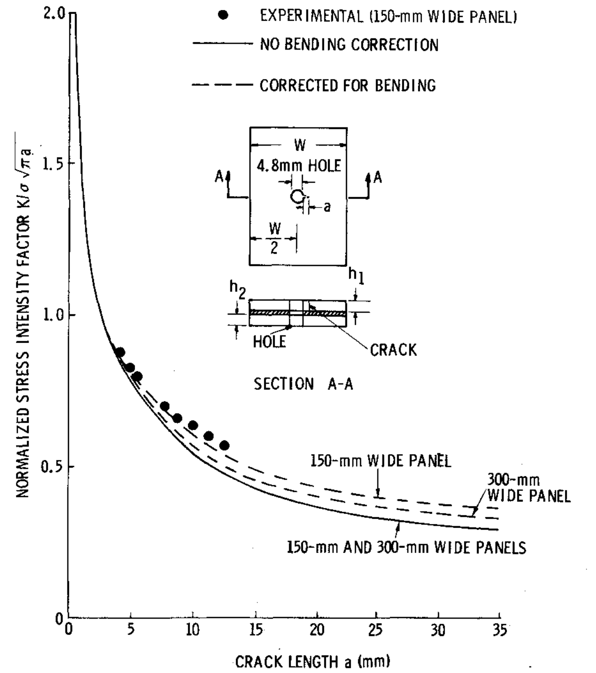

Fig. 9 Variation of stress intensity factors with crack length a for a two-ply, adhesively bonded structure with a crack emanating from a hole.

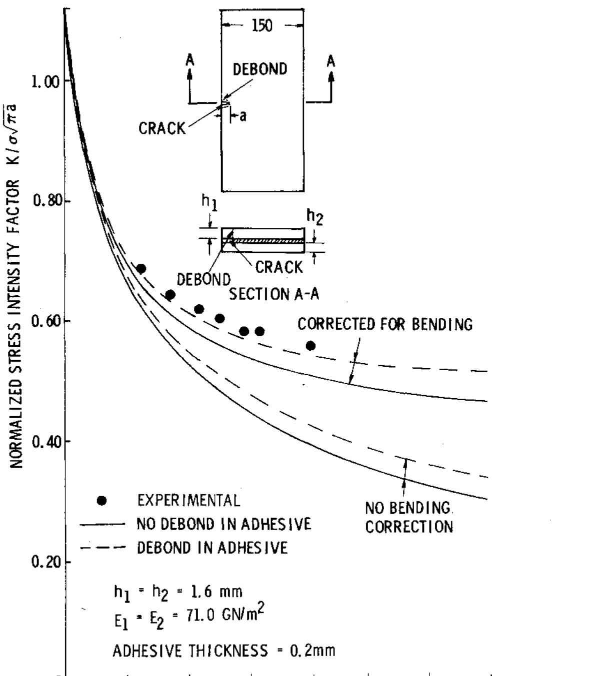

Fig. 8 Variation of stress intensity factors with crack length a for a two-ply, adhesively bonded structure with an edge crack.

Comparison of Experimental and Bending Corrected Stress Intensity Factors

For the two-ply bonded structure shown in Fig. 1, the stress intensity factors corrected for bending using Eq. (19) are shown in Fig. 5. It is seen that the experimental stress intensity factors agree very well with the analytical stress intensity factors corrected for the influence of bending. The finite-element determined stress intensity factors, corrected for the infleucne of bending, showed good correlation with experimental stress intensity factors obtained for various crack and panel geometries.[6,15]

Stress Intensity Factors for Various Crack and Panel Geometries

Stress intensity factors were obtained using the finite-element method for various crack and panel geometries. In computing these stress intensity factors, the adhesive shear modulus of 0.414 GN/m^2 (60,000 psi), corresponding to the FM 73 adhesive properties, was used.

For the two-ply bonded structure shown in Fig. 1, the stress intensity factors, assuming an elliptical debond in the adhesive with $b/a = 0.1$, are shown for two panel widths in Fig. 7. The stress intensity factors corrected for the influence of bending are also shown in the figure.

The stress intensity factors for a two-ply, bonded structure with an edge crack are shown in Fig. 8 for the cases of no debond, and an elliptical debond with $b/a = 0.1$ in the adhesive. The figure also shows experimental stress intensity factors. The analytical stress intensity factors, corrected for the influence of bending, agree well with the experimental results.

The stress intensity factors for a crack emanating from a central hole in a two-ply bonded structure (for two panels 150 and 300 mm wide) are shown in Fig. 9. These stress intensity factors without bending correction are identical, as the influence of the finite boundaries for the crack lengths examined is negligible. For the 150-mm-wide panel, the stress intensity factors corrected for the influence of bending agree well with the experimental results.

Parametric Study of Stress Intensity Factors for a Two-Ply, Bonded Structure with a Center Crack in One Layer

A parametric study was conducted to evaluate the influence of debond size, and adhesive and adherend properties on stress intensity factors for the two-ply bonded structure shown in Fig. 1. The integral equation formulation has shown that the influence of adhesives on stress intensity factors can be studied by the parameter h_a/μ_a (ratio of adhesive thickness to shear modulus). Similarly, the influence of the cracked and sound layer parameters can be studied by the parameter hE (where h is the thickness and E is the modulus of the given

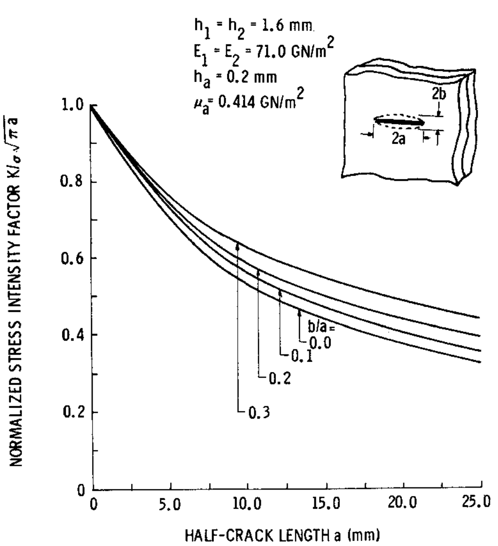

Fig. 10 Influence of debond size on stress intensity factors in a two-layer, center-cracked bonded structure.

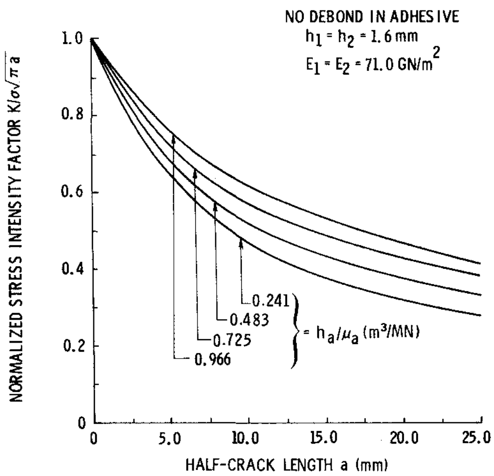

Fig. 11 Influence of adhesive parameter h_a/μ_a on stress intensity factors in a two-layer, center-cracked bonded structure.

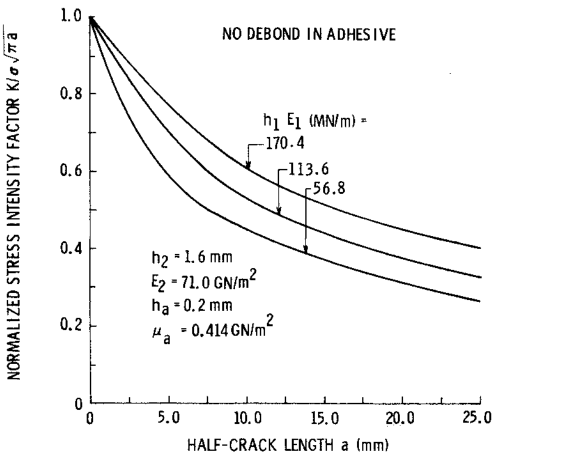

Fig. 12 Influence of cracked-layer parameter $h_l E_l$ on stress intensity factors in a two-layer, center-cracked bonded structure.

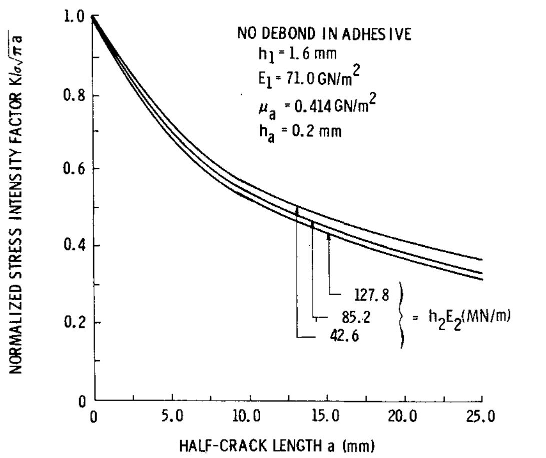

Fig. 13 Influence of sound layer parameter h_2E_2 on stress intensity factors in a two-layer, center-cracked bonded structure.

layer) and Poisson's ratio, ν. In the studies, Poisson's ratio, ν, was assumed to be 0.33 for metallic layers.

The influence of debond size on the stress intensity factors for various crack lengths is shown in Fig. 10. These stress intensity factors have been obtained for no debond, and an elliptical debond. The end of the major axis of the debond is assumed to coincide with the leading edge of the crack. It is seen that an increase in debond size increases the stress intensity factors, due to less load transfer to the sound layer.

The variation of the stress intensity factors with half-crack length a for various values of the adhesive parameter h_a/μ_a is shown in Fig. 11. An increase in the adhesive thickness, or a decrease in the shear modulus, causes an increase in the intensity factors due to less load transfer taking place to the uncracked layer.

Figure 12 shows the variation of stress intensity factors with half-crack length a for various values of cracked layer parameter $h_l E_l$. An increase in $h_l E_l$ causes an increase in the stress intensity factors due to the reduced stiffening influence of the sound layer.

The influence of the sound layer parameter on the stress intensity factors is shown in Fig. 13. An increase in $h_2 E_2$ causes a reduction in the stress intensity factors due to an increased stiffening effect. The reduction in stress intensity factors is dependent on the crack length.

Conclusions

Based on the preceding studies, the following conclusions are reached:

1) The analytical techniques (finite-element and mathematical) have been developed for calculating stress intensity factors that agree well with experimental results.

2) The results of analytical stress intensity factors obtained by finite-element and mathematical methods for adhesively bonded structures will be the same, if proper boundary conditions are incorporated.

3) A crack in one layer of a multilayered structure may induce bending, which will increase the stress in the cracked layer, thus the stress intensity factors. The influence of bending must be accounted for in the analysis.

4) A debond in the adhesive will cause less load transfer to the sound layer, hence an increase in stress intensity factors.

5) An increase in adhesive thickness, or a reduction in shear modulus, causes less load transfer to the sound layer, resulting in an increase in the stress intensity factors.

6) The influence of a sound or cracked layer on the stress intensity factors can be studied by varying two parameters—Poisson's ratio and the product of the modulus and thickness.

7) An increase in the cracked layer thickness or elastic modulus will cause an increase in the stress intensity factors.

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